WEEKLY TEST TARGET - JEE - TEST - 27
SOLUTION Date 01-12-2019

[PHYSICS]

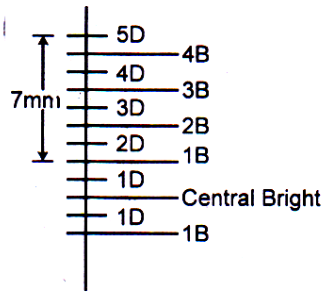
1. $$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2 - \left(\sqrt{I_1}-\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2 + \left(\sqrt{I_2}-\sqrt{I_2}\right)^2}$$

$$= \frac{I_1}{I_1} \times \frac{\left(1+\sqrt{\frac{I_2}{I_1}}\right)^2 - \left(1-\sqrt{\frac{I_2}{I_1}}\right)^2}{\left(1+\sqrt{\frac{I_2}{I_1}}\right)^2 + \left(1-\sqrt{\frac{I_2}{I_1}}\right)^2} = \frac{(1+2)^2-(1-2)^2}{(1+2)^2+(1-2)^2} = \frac{8}{10} = \frac{4}{5}$$

2. There are three and a half fringes from first maxima to fifth minima as shown.

$$\Rightarrow \quad \beta = \frac{7mm}{3.5} = 2mm$$

$$\Rightarrow \quad \lambda = \frac{\beta D}{d} = 600 \text{ nm.}$$



3. (B) For 100th max.

$d \sin\theta = 100\,\lambda$

$$\sin\theta = \frac{100 \times 5000 \times 10^{-9}}{1 \times 10^{-3}} = \frac{5 \times 10^{-4}}{10^{-3}} = 0.5 = \frac{1}{2}$$

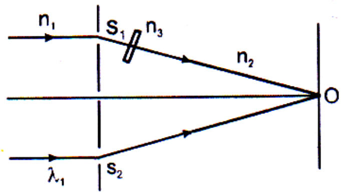
$$\therefore \quad y = D\tan\theta \quad = 1 \times \tan 30 \quad = \frac{1}{\sqrt{3}}$$

4. At path difference $\frac{\lambda}{6}$, phase difference is $\frac{\pi}{3}$

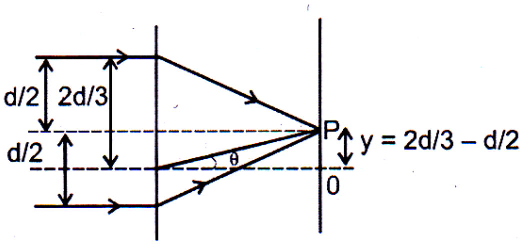
$I = I_0 + I_0 + 2I_0 \cos\frac{\pi}{3} = 3I_0$ $I_{max} = 4I_0$

So the required ratio is $\frac{3I_0}{4I_0} = 0.75$

5. $\frac{2\pi}{n_1\lambda_1}(n_3 - n_2)\,t$



6.



we know that P will be the central maxima (at which path difference is zero)

Now $OP = \frac{d}{2} - \frac{d}{3} = \frac{d}{6}$

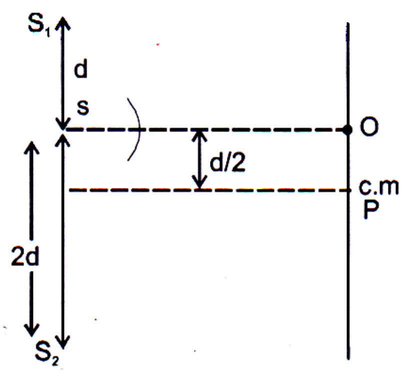
7.

, The 2 sources are.

As O is a maxima, Hence OP = β.

$\Rightarrow \quad \frac{d}{2} = \frac{\lambda.D}{(3d)};$

get $\quad \lambda = \frac{3d^2}{2D}$



8. For dark fringe

Path difference $= (2n-1)\frac{\lambda}{2}$

9.

According to lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For given concave lens,

$$R_1 = -3 \text{ cm} \quad \text{and} \quad R_2 = -4 \text{ cm}$$

$$\therefore \quad \frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{-3} + \frac{1}{4}\right)$$

or $$\frac{1}{v} - \frac{1}{(-12)} = (1.5 - 1)\left(\frac{-4+3}{12}\right)$$

or $$\frac{1}{v} + \frac{1}{12} = 0.5 \times \frac{-1}{12} = \frac{-1}{24}$$

or $$\frac{1}{v} = -\frac{1}{24} - \frac{1}{12} = \frac{-1-2}{24} = -\frac{1}{8}$$

or $$v = -8 \text{ cm.}$$

10.

Here, $f_a = 0.15$ m, $\mu_g = 3/2, \mu_w = 4/3$

According to lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \qquad \text{where } \mu = \frac{\mu_l}{\mu_m}$$

$$\frac{1}{f_a} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left[\frac{(3/2)}{1} - 1\right]C, \text{ where } C = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_a} = \frac{C}{2} \qquad \ldots \text{(i)}$$

Also, $$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left[\frac{(3/2)}{(4/3)} - 1\right]C$$

$$\frac{1}{f_w} = \frac{C}{8} \qquad \ldots \text{(ii)}$$

From eqns. (i) and (ii), we get;

$$\frac{f_w}{f_a} = \frac{C}{2} \times \frac{8}{C} = 4$$

or $$f_w = 4\ f_a = 4 \times 0.15 \text{ m} = 0.6 \text{ m.}$$

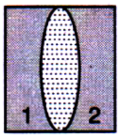
11.

As shown in the figure, the system is equivalent to combination of three thin lens in contact.

$$\therefore \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

By lens maker's formula,

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} - \frac{1}{25}\right) = \frac{-1}{50}$$



$$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{25} + \frac{1}{20}\right) = \frac{3}{100}$$

$$\frac{1}{f_3} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-20} - \frac{1}{\infty}\right) = -\frac{1}{40}$$

$$\frac{1}{f} = \frac{1}{5}\left[-\frac{1}{10} + \frac{3}{20} - \frac{1}{8}\right]$$

$$= \frac{1}{5}\left[\frac{-8 + 12 - 10}{80}\right] = \frac{1}{5}\left[\frac{-6}{80}\right]$$

or $$f = -\frac{400}{6}\text{ cm} = -66.6\text{ cm}$$

Hence, the system behaves as a concave lens of focal length 66.6 cm.

12.

Here, $f_1 = 20$ cm, $f_2 = 25$ cm.

The effective power of the combination is,

$$P = P_1 + P_2 = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{100}{20} + \frac{100}{25} \quad \left(\because P\text{ (in dioptre)} = \frac{100}{f\text{ (in cm)}}\right)$$

$$= 5\text{D} + 4\text{D} = 9\text{D}.$$

13.

Here, $v = +15$ cm, $u = +(15 - 5) = +10$ cm.

According to lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{15} - \frac{1}{10} = \frac{1}{f}$$

$$f = -30\text{ cm}.$$

14.

Given: $u + v = 80$ cm ...(i)

and $m = \left|-\frac{v}{u}\right| = +3$ ...(ii)

The image is inverted, $v = 3u$

$\therefore$ $u + 3u = 80$ cm or $u = 20$ cm

$\therefore$ $\frac{1}{20} + \frac{1}{60} = \frac{1}{f}$ or $f = 15$ cm

Object is between F and $2F$ ($u = 20$ cm). So, real, inverted, magnified image is formed beyond $2F$ (80 cm > 30 cm).

$\therefore$ $v > 2f$.

15.

For a plano-convex lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R}\right)$$

Given: $\mu = 1.5$, $f = 20$ cm

$\therefore$ $R = (\mu - 1)f = (1.5 - 1)20 = 10$ cm.

16.

Focal length of each plano-convex lens = 24 cm

for the liquid lens, $\frac{1}{f} = (\mu - 1)\left(\frac{-2}{12}\right) = \frac{1-\mu}{6}$

$\therefore \quad -\frac{1}{60} = \frac{1}{12} + \frac{1-\mu}{6}$

or $\quad \mu = 1.6.$

17.

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

or $\quad \frac{1}{f} = \frac{1}{0.2} + \frac{1}{0.2} - \frac{0.5}{(0.2)(0.2)}$

or $\quad 1/f = 5 + 5 - 0.5 \times 5 \times 5$

or $\quad 1/f = 10 - 12.5 = -2.5$

or $\quad f = -(1/2.5) = -0.4$ m.

18.

19.

Lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(where, $R_2 = \infty, R_1 = 0.3$ m)

$\therefore \quad \frac{1}{f} = \left(\frac{5}{3} - 1\right)\left(\frac{1}{0.3} - \frac{1}{\infty}\right)$

$$\frac{1}{f} = \frac{2}{3} \times \frac{1}{0.3}$$

or $\quad f = -0.45$ m.

20.

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \text{or} \quad \frac{1}{-20} + \frac{1}{v} = \frac{1}{20}$$

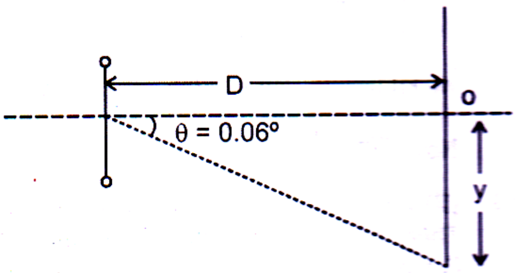
$\therefore \quad v = 10$ cm

$$m = \frac{v}{u} = \frac{h_2}{h_1}$$

or $\quad \frac{10}{20} = \frac{h_2}{2\text{ mm}}$ or $h_2 = 1$ mm.

21.

Say 'n' fringes are present in the region shown by 'y'



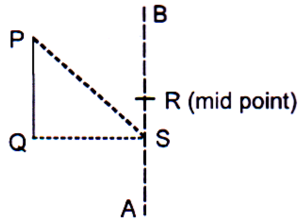
$\Rightarrow \quad y = n\beta = \frac{n.\lambda D}{d}$

$\Rightarrow \quad \frac{y}{D} \approx \tan(0.06^\circ) \approx \frac{0.06 \times \pi}{180} = \frac{n\lambda}{d}$

$\Rightarrow \quad n = \frac{10^3 \times \pi}{180} \times 0.06 = \frac{\pi}{3} > 1.$

Hence; only one maxima above and below point O. So total 3 bright spots will be present (including point 'O' i.e. the central maxima).

22.



Lets take any general point S on the line AB.
Clearly: for any position of S on line AB; we have for Δ PQS:
PQ + QS > PS {in any triangle sum of 2 sides is more then the third side}
⇒ $PS - QS < 3\lambda$.
As PS – QS represents the path difference at any point on AB ⇒ it can never be more than 3λ. Now minimas occur at.

$\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$ only.

so 3 minimas below R (mid point of AB) and 3 also above R.

23. For maximum intensity on the screen,

$$d \sin\theta = n\lambda$$

or $$\sin\theta = \frac{n\lambda}{d} = \frac{(n)(2000)}{(7000)}$$

$$= \frac{n}{3.5}$$

Since, $\sin\theta \not> 1$

∴ $n = 0, 1, 2, 3$ only.

Thus, only seven maximas can be obtained on both sides of the screen.

24.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \text{...(i)}$$

Given, $f = 10$ cm (as lens is converging)
$u = -8$ cm (as object is placed on left side of the lens)
Substituting these values in eqn. (i), we get;

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-8} \quad \text{or} \quad \frac{1}{v} = \frac{1}{10} - \frac{1}{8}$$

or $$\frac{1}{v} = \frac{8-10}{80}$$

or $$v = \frac{80}{-2} = -40 \text{ cm}$$

Hence, magnification produced by the lens,

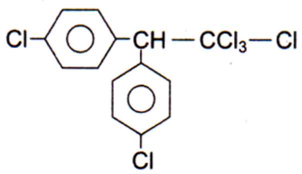
$$m = \frac{v}{u} = \frac{-40}{-8} = 5.$$

25.

We know that when convex lens is made of three different materials, then it has three refractive indices and therefore three focal lengths. Hence, number of images formed by the lens will be three.

[CHEMISTRY]

26. **(a)**



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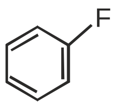
27. **(d)** Its vapours are non-inflammable (i.e. do not catch fire). Hence used as fire extinguishers under the name pyren.

28. **(d)** SN_1 reaction gives racemic mixture with slight predominance of that isomer which corresponds to inversion because SN_1 also depends upon the degree of 'shielding' of the front side of the reacting carbon.

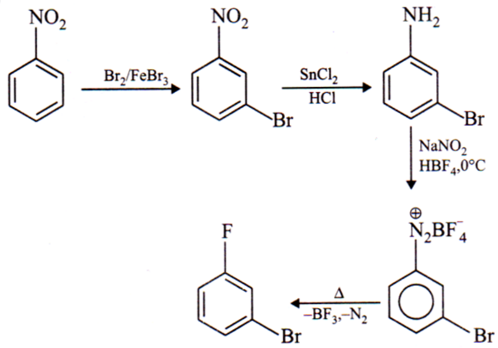
29. **(b)** Chloroform is oxidised to a poisonous gas, phosgene ($COCl_2$) by atmospheric gas.
$CHCl_3 + O \longrightarrow COCl_2 + HCl$

30. **(b)** Vinyl chloride shows resonance,
$$CH_2{=}CH{-}\ddot{C}l\!: \longleftrightarrow :\bar{C}H_2{-}CH{=}\overset{+}{\ddot{C}l}\!:$$
Due to resonance C—Cl bond has partial double bond character so bond is broken with difficulty.

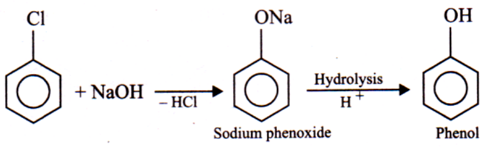
31. **(c)** $C_6H_5.CH_2Br \xrightarrow{Mg/ether} C_6H_5.CH_2MgBr$
$$\xrightarrow{H_3O^+} C_6H_5.CH_3 + Mg\begin{matrix}Br\\OH\end{matrix}$$

32. (A) Fluoro benzene 

33. **(b)** Sandmeyer's reaction.

34. **(d)**


35. **(c)** Gammaxane

36. **(c)** 

37. **(d)** Resonance stablization and the hybridisation of C attached to halide cis sp^2.

38. **(c)** With ethoxide base, most substituted alkene (I) is formed as the major product. In the formation of (II), $C_2H_5O^-$ takes proton from less hindered β-carbon, hence less activation energy and greater rate of reaction although stability of product determines it content at equilibrium. Also, since E2 reaction is an elementary reaction in which halogen leaves in the rate determining step, iodide leaves most easily and fluoride with maximum difficulty.

39. **(b)** $CH_3 - \underset{\displaystyle |\atop Br}{CH} - CH_2 - CH_3 + \underset{(alc)}{KOH} \xrightarrow{\text{Saytzeff's rule}}$

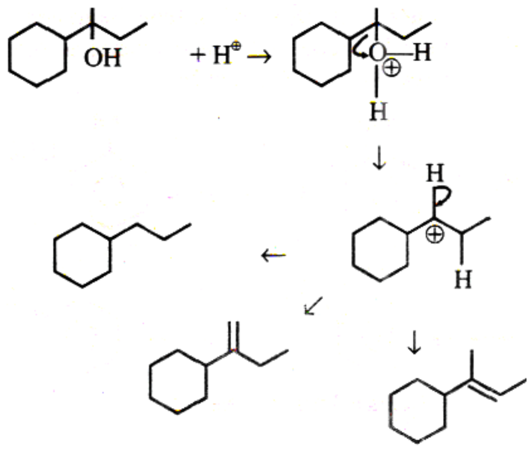
40. $CH_3CH_2OH \xrightarrow{PBr_3} CH_3CH_2Br \xrightarrow{alc.KOH} CH_2 = CH_2$

$\downarrow H_2SO_4$

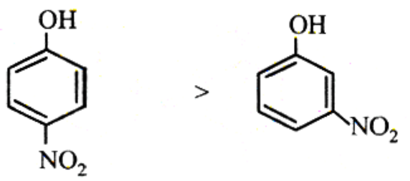
$CH_3CH_2OH \xleftarrow[heat]{H_2O} CH_3 - CH_2 - HSO_4$

41. Electron withdrawing $- NO_2$ group has very strong –I and –R effects so, compound 3 will be most acidic.

42.

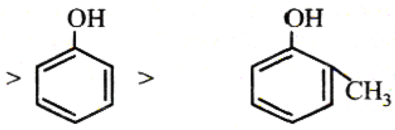


43.



(iv) (iii)

(– I and – M effects, both increase acidity) (only – I effect)



(i) (ii)

(+ I effect of CH_3 group decreases acidity)

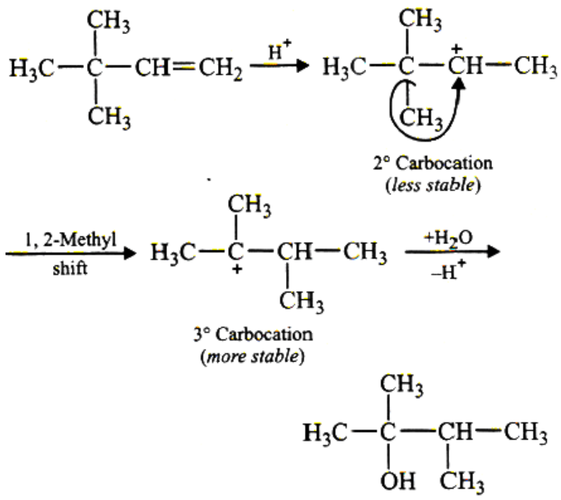
∴ Correct choice : (b)

44. $CH_3 - CH_2 - CH = CH_2 \xrightarrow[\text{(Peroxide effect)}]{HBr/H_2O_2}$

$\underset{(Y)}{CH_3 - CH_2 - CH_2 - CH_2Br}$

$\downarrow$

$\underset{(Z)}{CH_3 - (CH_2)_3 - O - CH_2 - CH_3}$

45. 

46.

47.

48. $(CH_3)_2 - C = \underset{|}{CH} \xrightarrow[\text{hydrogenation}]{\text{Catalytic}} (CH_3)_2 - CH - \underset{|}{CH_2}$ (with CH_3 below the CH on the left and below the CH_2 on the right)

49.

50. The hydrogen atom which is attached to triple bond is acidic

[MATHEMATICS]

51.

Resultant of $\vec{a}$ and $\vec{b}$ means $\vec{a}+\vec{b}$.

Here $\vec{a}+\vec{b} = (2\hat{i}+\hat{j}+\hat{k}) + (\hat{i}+2\hat{j}+3\hat{k})$

$= 3\hat{i}+3\hat{j}+4\hat{k}$

$\therefore |\vec{a}+\vec{b}| = \sqrt{3^2+3^2+4^2} = \sqrt{9+9+16}$

$= \sqrt{34}$.

52.

$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$

$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} - \overrightarrow{ED}$

$= \vec{a} + \vec{b} + \vec{c} - \overrightarrow{AB} = \vec{a} + \vec{b} + \vec{c} - \vec{a}$.

53.

If P.V. of the fourth vertex be $\vec{v}$, then

$$\frac{\vec{v}+(\hat{i}+3\hat{j}+5\hat{k})}{2} = \frac{(\hat{i}+\hat{j}+\hat{k})+(7\hat{i}+9\hat{j}+11\hat{k})}{2}$$

($\because$ diagonals bisect each other)

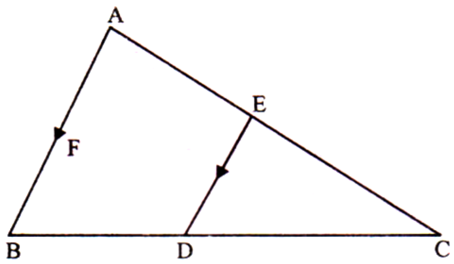
54.

P.V. of P $= \dfrac{3\vec{b}+\vec{a}}{3+1}$ and

P.V. of Q $= \dfrac{\text{P.V. of A} + \text{P.V. of P}}{2}$

55.

From geometry, we know that [ED] is equal to half of [AB] in length and is parallel to [AB], therefore, $\overrightarrow{AB} = 2\,\overrightarrow{ED}$.



56.

$$\overrightarrow{AC} - \overrightarrow{BD} = (\overrightarrow{AB} + \overrightarrow{BC}) - (\overrightarrow{BC} + \overrightarrow{CD})$$

$$= \overrightarrow{AB} - \overrightarrow{CD} = \overrightarrow{AB} + \overrightarrow{DC}$$

$$= \overrightarrow{AB} + \overrightarrow{AB} = 2\,\overrightarrow{AB}$$

57.

Let D be the mid-point of segment [BC], then

$$2\,\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AD} = \frac{1}{2}\left\{(3\hat{i}+4\hat{k}) + (5\hat{i}-2\hat{j}+4\hat{k})\right\}$$

$$= 4\hat{i} - \hat{j} + 4\hat{k}$$

Hence required length of the median

$$= |\,AD\,| = \overrightarrow{AD}$$

$$= \sqrt{4^2 + (-1)^2 + 4^2} = \sqrt{33}.$$

58.

$$\left(\frac{1}{8}\hat{i} - \frac{3}{8}\hat{j} + \frac{1}{4}\hat{k}\right)\cdot(2\hat{i}+4\hat{j}+5\hat{k})$$

$$= \frac{2}{8} - \frac{12}{8} + \frac{5}{4} = 0.$$

59.

$$\vec{a}\times\vec{b} = \vec{b}\times\vec{c} \Rightarrow \vec{a}\times\vec{b} - \vec{b}\times\vec{c} = \vec{0}$$

$$\Rightarrow \vec{a}\times\vec{b} + \vec{c}\times\vec{b} = \vec{0} \Rightarrow (\vec{a}+\vec{c})\times\vec{b} = \vec{0}$$

$\Rightarrow$ either $\vec{a}+\vec{c} = \vec{0}$ or $\vec{a}+\vec{c}$ is parallel to

60.

$$|\vec{a}+\vec{b}|<1 \Rightarrow |\vec{a}+\vec{b}|^2<1$$

$$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})<1$$

$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta<1$$

$$\Rightarrow 1+1+2\cos\theta<1$$

$$\Rightarrow 2\cos\theta<-1 \Rightarrow \cos\theta<-\frac{1}{2}$$

$$\Rightarrow -1\le\cos\theta<-\frac{1}{2}$$

$(\because -1\le\cos\theta\le 1 \text{ for all } \theta)$

$$\Rightarrow \cos\pi\le\cos\theta<\cos\frac{2\pi}{3} \Rightarrow \pi\ge\theta>\frac{2\pi}{3}.$$

61.

$$\frac{(\hat{i}-2\hat{j}+\hat{k})\cdot(4\hat{i}-4\hat{j}+7\hat{k})}{\sqrt{4^2+(-4)^2+7^2}}$$

$$=\frac{4+8+7}{\sqrt{81}}=\frac{19}{9}.$$

62.

$$\vec{\alpha}\times\vec{\beta}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ -1 & 2 & -4\end{vmatrix}=-10\hat{i}+9\hat{j}+7\hat{k}$$

and $$\vec{\alpha}\times\vec{\gamma}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ 1 & 1 & 1\end{vmatrix}=4\hat{i}-3\hat{j}-\hat{k}.$$

63.

$$|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$$

iff $|\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2$,

i.e., iff $(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$,

i.e., iff $4\vec{a}\cdot\vec{b}=0$, *i.e.*, iff $a\perp\vec{b}$.

64.

$$\vec{a}+\vec{b}+\vec{c}=\vec{0} \Rightarrow |\vec{a}+\vec{b}+\vec{c}|=0$$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2=0$$

$$\Rightarrow (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})=0$$

$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2$$

$$+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0\cdot$$

65.

$$\vec{a}+\vec{b}+\vec{c}=\vec{0} \Rightarrow \vec{b}+\vec{c}=-\vec{a}$$

$$\Rightarrow |\vec{b}+\vec{c}|=|-\vec{a}|$$

$$\Rightarrow |\vec{b}+\vec{c}|^2=|-\vec{a}|^2$$

$$\Rightarrow (\vec{b}+\vec{c})\cdot(\vec{b}+\vec{c})=|\vec{a}|^2$$

$$\Rightarrow b^2+c^2+2\vec{b}\cdot\vec{c}=a^2$$

$$\Rightarrow b^2+c^2+2bc\cos\theta=a^2.$$

66.

Let $\vec{b}=(x, y, z)$, then

$$\vec{a}\cdot\vec{b}=0 \Rightarrow x-y+z=0 \qquad ...(1)$$

Also, $\vec{a}\times\vec{b}=\vec{c}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ x & y & z \end{vmatrix} = -2\hat{i}-\hat{j}+\hat{k}$$

$$\Rightarrow (-z-y)\hat{i}+(x-z)\hat{j}+(y+x)\hat{k}$$

$$=-2\hat{i}-\hat{j}+\hat{k}$$

$$\Rightarrow -z-y=-2,\ x-z=-1,\ y+x=1 \qquad ...(2)$$

From (1) and (2), we obtain

$x=0, y=1, z=1.$

$\therefore \vec{b}=(0,1,1)$.

67.

Since $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, therefore,

a unit vector at right angles to $\vec{a}$ and $\vec{c}$

= *a* unit vector at right angles to $\vec{a}$ and $\vec{b}$

$$=\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} \text{ or } -\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}.$$

68.

$$|\vec{u}\times\vec{v}|=|(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})|$$

$$=2|\vec{a}\times\vec{b}| \quad (\because \vec{a}\times\vec{a}=\vec{b}\times\vec{b}=\vec{0})$$

and $|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2$

$$=(ab\sin\theta)^2+(ab\cos\theta)^2=a^2b^2,$$

θ being the angle between $\vec{a}$ and $\vec{b}$

$$\Rightarrow |\vec{a}\times\vec{b}|=\sqrt{a^2b^2-(\vec{a}\cdot\vec{b})^2}$$

$$=\sqrt{2^2 2^2-(\vec{a}\cdot\vec{b})^2}.$$

69.

Given $\vec{a} = \overrightarrow{BC}$, $\vec{b} = \overrightarrow{CA}$, $\vec{c} = \overrightarrow{AB}$,
therefore,

$$\vec{a} + \vec{b} + \vec{c} = \overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB} = \vec{0}$$

$\Rightarrow \quad \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow \quad (\vec{a} + \vec{b}) \times \vec{c} = (-\vec{c}) \times \vec{c}$

$\Rightarrow \quad \vec{a} \times \vec{c} + \vec{b} \times \vec{c} = \vec{0}$

$\Rightarrow \quad \vec{b} \times \vec{c} = -\vec{a} \times \vec{c}$

$\Rightarrow \quad \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

Similarly, $\vec{c} \times \vec{a} = \vec{a} \times \vec{b}$

Hence, $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

70.

Since $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular, therefore, $(\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 4\vec{b}) = 0$

$\Rightarrow \quad 5\vec{a} \cdot \vec{a} - 8\vec{b} \cdot \vec{b} + 10\vec{b} \cdot \vec{a} - 4\vec{a} \cdot \vec{b} = 0$

$\Rightarrow \quad 5 - 8 + 6\vec{b} \cdot \vec{a} = 0$

$\Rightarrow \quad \vec{b} \cdot \vec{a} = \frac{3}{6} = \frac{1}{2} \Rightarrow \cos\theta = \frac{1}{2}$,

where θ is the angle between $\vec{a}$ and $\vec{b}$.

71.

$(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 144$

$\Rightarrow \quad |\vec{a}|^2 |\vec{b}|^2 = 144$ (Legrange's Identify)

$\Rightarrow \quad 4^2 |\vec{b}|^2 = 144$

$\Rightarrow \quad |\vec{b}|^2 = 9 \Rightarrow |\vec{b}| = 3$.

72.

Given

$$(\hat{i} + \hat{j} + \hat{k}) \cdot \frac{(2+b)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+b)^2 + 6^2 + 2^2}} = 1$$

$\Rightarrow \quad (2 + b)\,1 + 6 - 2 = \sqrt{(2+b)^2 + 40}$

$\Rightarrow \quad (6 + b)^2 = 44 + 4b + b^2$

$\Rightarrow \quad 8b = 8 \Rightarrow b = 1$.

73.

First, we note that

$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$$

$$= 3 + 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$$

$$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = \frac{1}{2}|\vec{a}+\vec{b}+\vec{c}|^2 - \frac{3}{2} \geq -\frac{3}{2}$$

$$\Rightarrow -2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) \leq 3 \qquad ...(1)$$

$$\therefore |\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$$

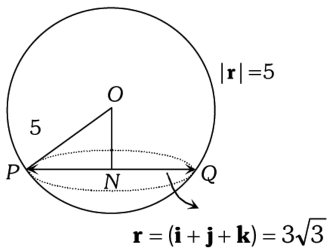
$$= 2|\vec{a}|^2 + 2|\vec{b}|^2 + 2|\vec{c}|^2 - 2[\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}]$$

$$= 2+2+2-2[\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}]$$

$\leq 6 + 3$ [Using (1)]

74. (a) We have $\overrightarrow{AP} = -3\mathbf{i} - \mathbf{j} + 10\mathbf{k}$

$\therefore |\overrightarrow{AP}| = \sqrt{9+1+100} = \sqrt{110}$



AN = Projection of $\overrightarrow{AP}$ on $6\mathbf{i}+3\mathbf{j}-4\mathbf{k}$

$$= \left|\frac{\overrightarrow{AP}.(6\mathbf{i}+3\mathbf{j}-4\mathbf{k})}{|6\mathbf{i}+3\mathbf{j}-4\mathbf{k}|}\right| = \left|\frac{-18-3-40}{\sqrt{61}}\right| = \sqrt{61}$$

75. (d) The Given lines are $\mathbf{r}_1 = \mathbf{a}_1 + \lambda\mathbf{b}_1$, $\mathbf{r}_2 = \mathbf{a}_2 + \mu\mathbf{b}_2$

Where $\mathbf{a}_1 = 4\mathbf{i} - 3\mathbf{j} - \mathbf{k}$; $\mathbf{b}_1 = \mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$

$\mathbf{a}_2 = \mathbf{i} - \mathbf{j} - 10\mathbf{k}$; $\mathbf{b}_2 = 2\mathbf{i} - 3\mathbf{j} + 8\mathbf{k}$

$$|\mathbf{b}_1 \times \mathbf{b}_2| = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix} = -11\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}$$

Now $[(\mathbf{a}_2 - \mathbf{a}_1)\ \mathbf{b}_1\ \mathbf{b}_2] = (\mathbf{a}_2 - \mathbf{a}_1).(\mathbf{b}_1 \times \mathbf{b}_2)$

$= (-3\mathbf{i} + 2\mathbf{j} - 9\mathbf{k})(-11\mathbf{i} + 6\mathbf{j} + 5\mathbf{k}) = 0$

Therefore, shortest distance $= \dfrac{[(\mathbf{a}_2 - \mathbf{a}_1)\ \mathbf{b}_1\ \mathbf{b}_2]}{|\mathbf{b}_1 \times \mathbf{b}_2|} = 0$.